Cylindrical Electromagnetic Concentrator with Radial and Tangential Constitutive Parameter Spatially Invariant

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Abstract — A type of electromagnetic concentrator with radial and tangential constitutive parameters as constants is proposed, and all the constitutive parameters are nonsingular and positive, which makes it possible to construct this kind of concentrator with two-dimensional metamaterials. The effects of loss and perturbations of parameters on the performance of the concentrator are also investigated. Finally, a simplified concentrator suitable for the case that is not sensitive to the omnidirectional scattering fields is proposed. This study provides a feasible way for 2D electromagnetic concentrator design.

I. INTRODUCTION

In the past few years, great interests have been attracted in optical transformation, which was first proposed by Pendry [1, 2]. By employing the form invariance property of Maxwell's equations in different spaces, optical transformation opens up many possibilities to control the behavior of electromagnetic fields [3-5]. Recently, the optical transformation theory has been applied to the electromagnetic (EM) concentrator, and some theoretical analyses, numerical simulations and parameter designs have been devoted to the EM concentrating devices [6, 7]. In order to simplify the complicated constitutive tensor of the EM concentrator, the cylindrical EM concentrator with radial and tangential (r and θ direction) constitutive parameters spatially invariant is proposed in this paper, only the axial constitutive parameter is the function of radius. Moreover, all the constitutive parameters of the concentrator are nonsingular and positive, which makes it feasible to be realized with two-dimensional metamaterials, and the constitutive tensor gotten is easier to be realized than that in [7]. It is believed that this research improves the flexibilities for the EM concentrator design, which possesses potential applications in optical microscope, solar cell and devices that high field intensities are needed.

II. DERIVATION

According to the optical transformation theory and the form invariance of the Maxwell's equations, the constitutive parameters of the concentrator can be expressed as:

$$\begin{aligned} &\varepsilon_r = \mu_r = 1/\varepsilon_\theta = 1/\mu_\theta = f(r)/rf'(r) \\ &\varepsilon_z = \mu_z = f'(r)f(r)/r \end{aligned} \quad (1)$$

where $f(r)$ is the transformation function between the original space and the transformed space. The optical transformation for the concentrator can be expressed that the region $r' \in [0, R_2]$ is compressed into the region $r \in [0, R_1]$, and the region $r' \in [R_2, R_3]$ is focused into the region $r \in [R_1, R_3]$. Here r and r' represents the radius of the transformed space and the original space, respectively. It can be noticed in equation (1) that the numerical value of ε_r and ε_θ is reciprocal, if one of them is set as a constant, the other can be also fixed as a constant. Suppose that:

$$\varepsilon_\theta = rf'(r)/f(r) = m_0 \quad (2)$$

By solving the ordinary differential equation above, and based on the boundary conditions of the transform function, the unknown coefficients can be solved, and the constitutive tensor can be expressed as:

$$\begin{aligned} &\varepsilon_{corr} = \varepsilon_{cor\theta} = 1 \\ &\varepsilon_{corz} = R_2^2/R_1^2 \\ &\varepsilon_{cir\theta} = 1/\varepsilon_r = m_0 \\ &\varepsilon_{cirz} = m_0 \left(r/R_3\right)^{2(m_0-1)} \end{aligned} \quad (3)$$

where $m_0 = \log_{R_3/R_1} R_3/R_2 < 1$, and the subscript *cor* and *cir* represents for the core region and the circular region, respectively. Moreover, the impedance of the concentrator at the outer boundary can be expressed as $Z\big|_{r=R_3} = \sqrt{\mu_\theta/\varepsilon_z} = 1$. It can be seen that the cylindrical EM concentrator is always impedance matched with the free space, which indicates minimized scattering fields of the concentrator. Next there are some full wave simulations based on constitutive parameters above.

III. SIMULATIONS AND DISCUSSIONS

First, lossless cases are calculated based on the finite element method. Here the geometry parameters are selected as $R_3=2R_2=4R_1=0.4$m, and the frequency is set at 2GHz. Fig. 1a shows the electric fields distribution of the concentrator. It can be seen that the electric fields are concentrated into the inner core region smoothly, and the fields outside are rarely disturbed. Furthermore, the normalized power flow of the electromagnetic fields are also calculated and shown in Fig. 1b. It can be seen that power flow is enhanced obviously in the inner core region. Through the transformation function, it can be observed that the enhancing ratio can be expressed as the ratio of R_2 to R_1, and enhancement theoretically diverges to infinity as R_1 tending to zero.

Second, it does make sense to investigate the lossy cases, for that the artificial metamaterials that are necessary to fulfill the material properties of the concentrator are inevitably lossy in practice. Here two typical loss tangents are selected, namely 0.001 and 0.1. The power flows of the concentrator are displayed in Fig. 2. It can be seen that the power flow of the two cases changes obviously. When loss tangent equals to 0.001, there is little change comparing with the lossless case. As for the case that loss tangent is 0.1, for that most of the energy is expended in the circular region, and the concentrator just collects the residual energy and focuses it into the core region. That's exactly what Fig. 2b depicts.

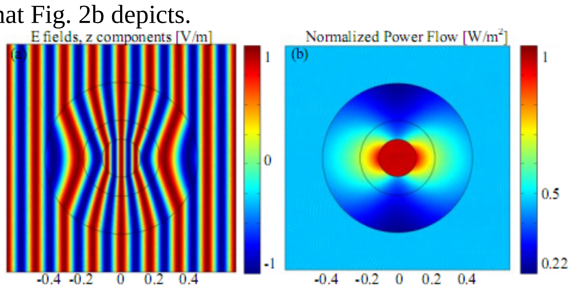

Fig. 1. (a) Electric fields distribution of the cylindrical EM concentrator. (b) Normalized power flow distribution of the concentrator.

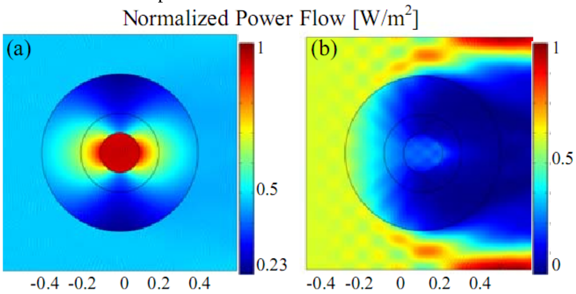

Fig. 2. Normalized power flow distributions of the concentrator (a) loss tangent equals to 0.001. (b) loss tangent equals to 0.1.

Furthermore, it can be seen in equation (3) that the constitutive tensor for the core region is anisotropic and homogeneous. If it can be transformed into the isotropic and homogeneous media, it will be easier in practical construction. For the cylindrical coordinate, the wave equation can be expressed as:

$$\left[\frac{1}{r\mu_z}\frac{\partial}{\partial r}\left(\frac{r}{\varepsilon_\theta}\frac{\partial}{\partial r}\right)+\frac{1}{r\mu_z}\frac{\partial}{\partial \theta}\left(\frac{1}{r\varepsilon_r}\frac{\partial}{\partial \theta}\right)+\left(\frac{\partial^2}{\partial z^2}+\omega^2\varepsilon_0\mu_0\right)\right]\varphi=0 \tag{4}$$

In our model, the ε_r and the ε_θ are all constants, if the product $\mu_z\varepsilon_\theta$ and $\mu_z\varepsilon_r$ are kept invariant, the propagating property of EM wave is kept invariant. So the constitutive tensor in equation (3) can be expressed as:

$$\varepsilon_{corr}=\varepsilon_{cor\theta}=\varepsilon_{corz}=R_2/R_1 \tag{5}$$

But this transformation results in the change of the impedance at the boundary $r=R_1$, in order to keep matching with the core region, the constitutive tensor of the circular region should be transformed into:

$$\begin{aligned}\varepsilon_{cirr}&=R_2/R_1 m_0\\ \varepsilon_{cir\theta}&=m_0R_2/R_1\\ \varepsilon_{cirz}&=m_0R_1\left(r/R_3\right)^{2(m_0-1)}/R_2\end{aligned} \tag{6}$$

Here the constitutive tensors of the core region and the circular region are gotten, and this type of concentrator is called simplified concentrator here. The distributions of electric fields and power flow of the simplified concentrator with the constitutive tensor above are also simulated and shown in Fig. 6. It can be seen in Fig. 6a that because the impedance is mismatched with the free space at the out boundary $r=R_3$, part of the incident wave are reflected inevitably, but the simplified concentrator still can focus the energy left into the core region, which is depicted in Fig. 6b. Moreover, through the selection of the numerical values of R_1, R_2 and R_3, the radial and the tangential constitutive parameters of the circular region can be larger than 1, only the z component of the constitutive tensor needs to be realized by metamaterials, so the simplified concentrator is easy to be constructed, and this type of concentrator is more suitable for the case that is not sensitive to the omnidierectional scattering fields.

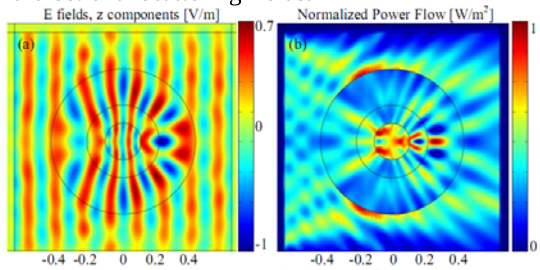

Fig. 3. (a) Electric fields distribution of the simplified EM concentrator. (b) Normalized power flow distribution of the simplified concentrator.

IV. CONCLUSIONS

In conclusion, a cylindrical EM concentrator with only axial permittivity/permeability spatially variant is proposed and designed theoretically based on the optical transformation theory, and then the electromagnetic characteristics of the concentrator are studied based on the finite element methods. Furthermore, the effects of loss and perturbations of parameters on the performance of the concentrator are also investigated. Finally, the concentrator with simplified constitutive parameters is proposed, which is suitable for the case that is not sensitive to the omnidirectional scattering fields. It is believed that the results are helpful to construct the cylindrical EM concentrator with existing metamaterials.

V. REFERENCES

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